

# Eliminating the Gravitational Constant $G$ from General Relativity and Cosmology

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## Abstract

The elimination from Newtonian gravitation and general relativity of the gravitational constant  $G$  as a dimensionless SI factor shows that the curvature of space is not constant and is highly flexible in locations of dense matter/energy. The elimination also constrains the cosmological modelling of galaxies and the evolution of the universe. The use of new double-adjusted SI units, instead of SI units, shows the simplification and better understanding of physics that could ensue

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## Introduction

The search for a way of understanding the Newtonian gravitational constant has been longstanding [1,2,3]. The importance of  $G$  is fundamental and ‘the poor precision of gravitational constant  $G$ , which is widely believed to define the ‘‘cube of theories’’ and the units of the future ‘‘theory of everything’’, does not allow to use  $G$  as a fundamental dimensional constant in metrology [4], shows the breadth of the issue. In the areas of general relativity and cosmology, for example, the value of  $G$  changes both the stiffness of the curvature of space and the evolution of the universe respectively.

The purpose of this paper is to show how it is possible to eliminate the gravitational constant  $G$  from Newtonian forms of equations and then to extend that to general relativity and show its effect on cosmology.

Previous work [1,2,3] has been based on the assumption that  $G$  is related only to mass in some way. The difference here is that both mass and distance are affected, and in equal and opposite ways. However, it is possible to understand the properties’ correct relationship only once SI units have been suitably adjusted to be internally consistent [5].

The starting point is to show how the change of the mass and distance Planck properties in SI units can become new Double-Adjusted SI Units (DASI). The main adjustment factor is  $\sqrt{G}$  which affects the mechanical side of SI units, whilst the other factor affects the size of the electronic charge and is not addressed here. DASI is integral to adjusting the relationships between properties. The result will be that  $G$  no longer appears in any formulae. The mathematics of the changes is very simple, but the implications are extremely significant.

## Significance and Objectives

The significance is in part that, whilst eliminating  $G$  does not change the size of any gravitational interactions, it does serve to show, for example, that the curvature of space is not constant, being highly flexible in locations of high energy density, and that the equal strength of gravitation and charge fields is hidden by SI units [5].

The analysis is bolstered by a novel method of extending the use of simple dimensions of properties and units beyond, for example, MLT (mass, length and time) into their underlying ‘dimensionality’ [5].

This use of dimensionality shows that the relationship between the energy momentum tensor  $T^{\mu\nu}$  [6] and the Einstein tensor  $G^{\mu\nu}$  [7] is not a true constant. The elimination of  $G$  also constrains models of galaxy formation and the evolution of the universe.

The main objective is to underline the need eventually to adjust SI units by the two factors mentioned [5] to provide the internally consistent DASI replacement set. This logically implies other objectives which include the adoption formulae excluding  $G$  and the adjustment of the electromagnetic SI units including the value of the electronic charge by the factor  $\sqrt{10^7}$ .

## Newtonian Force Formula

The relevant substitutions here for SI properties are only for mass and distance using  $\sqrt{G}$ . The Planck-based SI units of Planck mass  $M_p$  and Planck distance  $R_p$ , become the DASI sized units  $M_*$  and  $R_*$ , where

$$M_* = \sqrt{G} M_p$$

$$R_* = R_p / \sqrt{G}$$

So that the Newtonian force equation for a stable orbit between two Planck sized bodies at Planck orbital radius changes from

$$GM_p M_p / R_p^2 = M_p v^2 / R_p \quad (\text{SI})$$

to  $M_* M_* / R_*^2 = M_* v^2 / R_* \quad (\text{DASI})$

This change does not affect how far apart the two bodies orbit or their masses, but does change what that distance and those masses are called.

### Dimensionalities

Each property can be assigned a ‘dimensionality’, which allows the correctness of formulae, such as laws of nature, to be confirmed. This is an extension to the idea that many properties can be dissected into ‘units’ of mass, length and time [8].

The relevant basic properties in terms of the dimensionality factor  $Y$  here are

Mass	$M = Y^1$
Length	$R = Y^{-3}$
Time	$T = Y^{-5}$
Velocity	$v = Y^2$
Gravitational constant	$G = Y^0$
Vector differential (del)	$\nabla = Y^{-3}$
Planck constant	$h = Y^0$
Mass density	$\rho = Y^{10}$
Energy density	$\epsilon = Y^{14}$
Force	$F = Y^8$

Two examples of how laws of nature conform to dimensionalities would be

$$\text{Force} = M_* M_* / R_*^2 = M_* v^2 / R_*$$

$$Y^1 Y^1 / (Y^{-3} Y^{-3}) = Y^1 Y^2 Y^2 / Y^{-3} = Y^8$$

And Angular momentum  $= h = M_* v R_*$

$$Y^0 = Y^1 Y^2 Y^{-3} = Y^0$$

$G$  Can be confirmed as a true constant like  $h$ , with a dimensionality of zero, since its units are

$$G = m^3 k g^{-1} s^{-2} = Y^{-9} Y^{-1} Y^{10} = Y^0$$

Only properties with dimensionality  $Y^0$  are true constants and in DASI  $M_* = \sqrt{hc}$ .

### Gravity

Considering the differential form of Gauss’s law for gravity [9], using the usual meanings for all symbols, each term initially in SI units, although no longer with a subscript, then

Gravity  $g(M_p, R_p)$

Mass density  $\rho(M_p, R_p)$

Del  $\nabla(R_p)$

Gives the usual formula

$$\nabla \cdot g = -4\pi G \rho$$

Substituting  $\sqrt{G}$  in the following way produces the DASI equivalents

Gravity  $g_* = \sqrt{G} g$

Mass density  $\rho_* = \sqrt{G} M_p / (R_p / \sqrt{G})^3 = \rho G^2$

Del  $\nabla_* = (R_p / \sqrt{G})^{-1} = \sqrt{G} \nabla$

So that now in DASI

$$\nabla_* \cdot g_* = -4\pi \rho_*$$

The change in  $g$  can be shown as ( $e_r$  is a dimensionless vector)

$$\begin{aligned} g &= -GM_p e_r / R_p^2 \\ &= -(\sqrt{G} M_p) e_r / (\sqrt{G} (R_p / \sqrt{G})^2) \\ &= -M_* e_r / (\sqrt{G} R_*^2) \\ &= g_* / \sqrt{G} \end{aligned}$$

Thus  $G$  is eliminated from Gauss’s law for gravity when expressed in DASI units. This is an example of how stretching/compressing the units of mass and distance, inversely with respect to each other, allows the symmetric basis for the maximal sizes of properties to emerge and so eliminates the need for many dimensionless ratios like  $G$ . The same could be done for Planck’s constant  $h$ , but that result does not help better understanding of relationships between properties.

### General Relativity

The equivalent argument can be used for the Riemann tensor [10] using the relativistic generalization of Poisson’s equation for a gravitational field [11], initially in SI units. Here vector notation is not used since only the dimensionality of the properties and size of the constant factor  $k$  between the energy momentum tensor  $T^{\mu\nu}$  and the Einstein tensor  $G^{\mu\nu}$  is of interest.

$$-\nabla \cdot (-\nabla \phi) = 4\pi G \rho$$

Where  $\phi$  is the gravitational potential given in SI by

$$\begin{aligned} \phi &= -GM_p / R_p \\ &= -\sqrt{G} M_p / (R_p / \sqrt{G}) \\ &= -M_* / R_* \\ &= \phi_* \end{aligned}$$

So that, converting to DASI

$$\begin{aligned} -\nabla \cdot (-\nabla \phi) &= 4\pi G \rho \\ -(\nabla_* / \sqrt{G}) \cdot (-\nabla_* / \sqrt{G}) \phi_* &= 4\pi G (\rho_* / G^2) \\ -\nabla_* \cdot (-\nabla_* \phi_*) &= 4\pi \rho_* \end{aligned}$$

### Energy-Momentum Tensor

The standard formula for the energy-momentum tensor  $T$ , with  $\epsilon$  as the energy density and  $u$  as the four-velocity vector, is

$$T^{\mu\nu} = \epsilon_0 u^\mu u^\nu$$

The dimensionality of  $\epsilon_0$  is  $Y^{14}$  and the dimensionality of  $u$  is  $Y^0$ , the latter since it is a generalized form of  $v/c$ , so  $T^{\mu\nu}$  is  $Y^{14}$  also. Since  $v$  does not change between SI and DASI, the only issue is how  $\epsilon_0$  changes. The relationship is

$$\begin{aligned} \epsilon_0 &= M_p c^2 / R_p^3 \\ &= G^{-2} \sqrt{G} M_p c^2 / (R_p / \sqrt{G})^3 \\ &= G^{-2} M_* c^2 / R_*^3 \\ &= \epsilon_{0*} / G^2 \end{aligned}$$

Which is the same as for the mass density relationship. This means that

$$T_*^{\mu\nu} = G^2 T^{\mu\nu}$$

### Riemann Curvature Tensor

Since the concern once again is only the dimensionalities of the properties and size of the Einstein gravitational constant  $k$  [12] the standard formula for the Riemann tensor in the Newtonian limit can be used here, initially again in SI units

$$R^{00} \approx -\nabla^2 \phi / c^2$$

This, expressed in DASI then becomes

$$R^{00} \approx -\nabla_*^2 \phi_*/(Gc^2) \\ \approx R_*^{00}/G$$

Which follows through then to the relativistic expression for  $R_*^{\mu\nu}$ . The dimensionality of  $R$  is  $Y^6$  since it is mass density  $\rho$  of  $Y^{10}$  dimensionality divided by  $c^2$  of  $Y^4$ .

Taking the Einstein field equation [13] in SI units

$$G^{\mu\nu} = R^{\mu\nu} = 8\pi GT^{\mu\nu}/c^4$$

Where  $k = 8\pi G/c^4$  and substituting DASI units makes the equation now

$$G_*^{\mu\nu} = R_*^{\mu\nu} = 8\pi T_*^{\mu\nu}/c^4$$

Which eliminates  $G$  from the equation. Note that due to the different dimensionalities of  $R$  and  $T$ , the relationship between them is not a true constant.

### The ‘Constants’ $k$ And $c$

The relationship  $k$  between the two expressions for energy has been called a ‘constant’ because  $c$  has been assumed to be a constant. Now  $k_*$  becomes

$$k_* = 8\pi/c^4$$

However, as seen from the dimensionality of velocity,  $c$  has dimensionality  $Y^2$  so is clearly not a constant. This is also supported by the dimensionalities of  $R$  and  $T$ , respectively  $Y^6$  and  $Y^{14}$ . These require  $k_*$  to have dimensionality of  $Y^{-8}$  to make the equation dimensionally correct.

The speed of light is really a quasi-constant in that it takes the value in SI units of  $2.998 \times 10^8 \text{ ms}^{-1}$  in empty space. But when close to a black hole, the speed of light may be close to zero, but that speed is still the maximum it can manage and is still  $c$ .

This means that the curvature of space is not constant and is not stiff. The value of  $k_*$  depends on the local value of  $c$ , which could make  $k_*$  tend towards infinity as  $c$  approaches zero. So the denser the local environment in terms of mass/energy the more flexible space becomes.

### Cosmology

In the search for solutions to cosmological issues surrounding the growth, composition or evolution of galaxies and the universe [14,15,16] it is no longer possible to propose models based on constant, alternate or changing values of  $G$ . The gravitational constant is required as one of two factors to adjust SI units into DASI units so that they become internally consistent and eliminate unnecessary ‘constants’ that appear in formulae between properties.

### Conclusion

By adjusting SI units to become DASI units, it is possible to eliminate  $G$  from all equations. This highlights that the curvature of space, represented by the Einstein gravitational constant, is not constant and is highly flexible in energy dense locations. The use of DASI units provides an internally consistent set of adjusted-Planck sized property values. The introduction of dimensionality simplifies the understanding of relationships between properties. Eliminating  $G$  also puts constraints on the modeling of galaxies and the universe.

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